# FURTHER MATHS
## 1. INDICES

Indices are numbers in the form

$$a^x$$

$a$ is the base and $x$ is the index
Examples of number in the above form are $2^3$; $s^{-2}$; $2^{\frac{1}{2}}$; etc

**Coverting Whole Number to Indices Form**
For specific purposes sometimes, numbers can be converted to indices

**Example 1.0**
Covert (a) 36 (b) 48 (c) 120 to indices form
Solution
$36 = 2 \times 3 \times 2 \times 3 = 2^2 \times 3^3$
$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

**Law of Indices**

1. $x^a \times x^b = x^{a+b}$
2. $x^a \div x^b = x^{a-b}$
3. $x^0 = 1$
4. $x^{-a} = \dfrac{1}{x^a}$
5. $(x^a)^b = x^{a\times b} = x^{ab}$
6. $x^{\frac{1}{2}} = \sqrt{x}$
7. $x^{\frac{1}{a}} = \sqrt[a]{x}$
8. $x^{\frac{a}{b}} = \sqrt[a]{x^b} = (\sqrt[a]{x})^b$
9. $(ab)^m = a^m b^m$
10. $\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$

**Example 1.1**

1. Simplify $10^2 \times 10^3$
   Solution
   $10^2 \times 10^3 = 10^{2+3} = 10^5$
2. Simplify $5y \times 4y^4$
   Solution
   $$\begin{aligned} 5y \times 4y^4 &= 5 \times 4 \times y \times y^4 \\ &= 20 \times y^{1+4} \\ &= 20 \times y^5 \\ &= 20y^5 \end{aligned}$$
3. Simplify $x^3 \div x^{-5}$
   Solution
   $$\begin{aligned} x^3 \div x^{-5} &= x^{3-(-5+} \\ &= x^{3+5} \\ &= x^8 \end{aligned}$$
4. Simplify $\dfrac{9 \times 10^9}{3 \times 10^3}$
   Solution
   $$\begin{aligned} \frac{9 \times 10^9}{3 \times 10^3} &= \frac{3^2 \times 10^9}{3 \times 10^3} \\ &= \frac{3^2}{3^1} \times \frac{10^9}{10^3} \\ &= 3^{2-1} \times 10^{9-3} \\ &= 3 \times 10^6 \end{aligned}$$

**Class Exercise 1.0**

1. Simplify the following
   (a) $19^0$
   (b) $r \times r^0 \times r^{-5}$
   (c) $c^7 \div c$
   (d) $m^8 \div m^5$
   (e) $\dfrac{24x^6}{8x^4}$
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**Example 1.2**
Simplify (a) $(a^3)^2$ (b) $(h^4)^{-5}$ (c) $(3m^4)^2$ (d) $(-4^4)^3$ (e) $-(c^5)^4$
Solution

(a) $(a^3)^2 = a^3 = a^6$

(b) $(h^4)^{-5} = h^{4\times -5} = h^{-20}$

(c) $(3m^4)^2 = 3^2 \times (m^4)^2 = 3^2 \times m^8 = 9m^8$

(d) $(-4^4)^3 = (-1)^3(e^4)^3 = -e^{4\times 3} = -e^12$

(e) $-(c^5)^4 = -c^{5\times 4} = -c^20$

**Class Exercise 1.1**
Simplify the following
(1) $(d^4)^3$ (2) $(e^3)^3$ (3) $(2^{-3})^2$ (4) $(2n^5)^3$
(5) $4(r^3)^2$ (6) $(-c^3)^2$ (7) $(mn^2)^4$ (8) $\dfrac{(-x^3)^2}{-x^4}$
(9) $\dfrac{a^6}{(-a)^4}$ (10) $\dfrac{(-c)^2 \times c^4}{(-c)^5}$

**Example 1.3**
Simplify the following (a) $9^{\frac{1}{2}}$ (b) $8^{\frac{1}{3}}$ (c) $8^{-\frac{2}{3}}$ (d) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$
Solution

(a) $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}} = 3$

(b) $8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2$

(c) $8^{-\frac{2}{3}} = (2^3)^{-\frac{2}{3}} = 2^{3\times-\frac{2}{3}} = 2^{-2}$

(d)
$$\begin{aligned} \left(\frac{16}{81}\right)^{-\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{-\frac{3}{4}} = \frac{(2^4)^{-\frac{3}{4}}}{(3^4)^{-\frac{3}{4}}} &= \frac{2^{-3}}{3^{-3}} \\ &= \frac{\frac{1}{2^3}}{\frac{1}{3^3}} \\ &= \frac{1}{2^3} \times \frac{3^3}{1} \\ &= \frac{27}{8} \end{aligned}$$

**Class Exercise 1.2**
Simplify the following

(a) $2a \times 3a^2$ (b) $2a \times (3a)^2$
(c) $27^{\frac{1}{3}}$ (d) $4^{1-2}$
(e) $\sqrt[3]{2^6}$ (f) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$
(g) $3^{\frac{1}{2}} \times 3^{-\frac{3}{2}}$ (h) $16^{-\frac{3}{4}}$
(i) $3^x \times 3^{-x}$ (j) $\dfrac{1}{3^{-2}}$
(k) $3^{n-1} \times 3^{1-n}$ (l) $125^{-\frac{2}{3}}$
(m) $0.04^{\frac{1}{2}}$ (n) $\left(\dfrac{1}{9}\right)^{-1}$
(o) $64^{-\frac{5}{6}}$ (p) $3a^{-2}$
(q) $\dfrac{75a^2b^{-2}}{5a^3b^{-3}}$

More exercises on New General Mathematics Eercise 1e page 18

## Indicial Equation

**Indicial Equation with Unknown Base**
**Example 1.4**
Solve for $x$ in

$$x^{\frac{1}{2}} = 2$$

Solution

$$x^{\frac{1}{2}} = 2^1$$

multiply both indexes by $\frac{2}{1}$

$$x^{\frac{1}{2}\times\frac{2}{1}} = 2^{1\times\frac{2}{1}}$$

$$x^1 = 2^2 \quad \Rightarrow \quad x = 4$$

**Example 1.5**
$2x^3 = 54$; Solve for $x$
Solution

$$2x^3 = 54$$

Dividing both sides by 2 we have

$$x^3 = 27$$

Multiply both indexes by $\frac{1}{3}$

$$x^{3\times\frac{1}{3}} = 27^{\frac{1}{3}}$$

$$x = (3^3)^{\frac{1}{3}} = 3$$

$$x = 3$$

**Example 1.6**
Solve for $x$ in $5x = 40x^{-\frac{1}{2}}$
Solution

$$\frac{x}{x^{-\frac{1}{2}}} = \frac{40}{5}$$

$$x^{1-(-\frac{1}{2})} = 8$$

$$x^{\frac{3}{2}} = 8$$

multiply both indexes by $\frac{2}{3}$

$$x^{\frac{3}{3}\times\frac{2}{3}} = 8^{\frac{2}{3}}$$

$$x = (2^3)^{\frac{2}{3}} = 2^2$$

$$x = 4$$

**Class Exercise 1.3**
Solve the following equations

(a) $x^{-\frac{1}{3}} = 3$ (b) $a^{-1} = 2$
(c) $a^{-2} = 9$ (d) $x^{-\frac{1}{2}} = 5$
(e) $n^{-\frac{2}{3}} = 9$ (f) $2r^{-3} = -16$

**Indicial Equation with unknown Index**
To be able to solve indicial equation with unknown index, consider the indicial equation law
If

$$a^x = a^y$$

then

$$x = y$$

**Example 1.7**
if $8^x = 32$; find $x$
Solution

$$8^x = 32$$

$$(2^3)^x = (2^5)$$

$$2^{3x} = 2^5$$

$$\Rightarrow \quad 3x = 5$$

$$\Rightarrow \quad x = \frac{5}{3}$$

**Class Exercise 1.4**
Solve the following equations

a. $9^x = 27$

b. $5^x = 25$

c. $4^{c-1} = 64$

d. $3^{2x-1} = 81$

# 2. LOGARITHMS

logarithms can be viewed as another way of expressing indices
Consider

$$10^2 = 100$$

This means that the number 10 multiplies itself twice to result to 100

Another way to write this is

$$\log_{10} 100 = 2$$

This means that the number of times 10 multiplies itself to result to 100 is 2

Infact if

$$a^x = b$$

then

$$\log_a b = x$$

Number in the form

$$\log_a b$$

are called logarithms
hence we have $\log_2 5$; $\log_7 11$ etc

**Example 2.0**
Evaluate $\log_2 32$

Solution
Let

$$x = \log_2 32$$

Then

$$\begin{aligned} 2^x &= 32 \\ 2^x &= 2^5 \\ \Rightarrow \quad x &= 5 \\ \therefore \quad \log_2 32 &= 5 \end{aligned}$$

**Example 2.1**
Evaluate $\log_9 27$
Solution
let

$$\begin{aligned} x &= \log_9 27 \\ \Rightarrow \quad 9^x &= 27 \\ (3^2)^x &= 3^3 \\ 3^{2x} &= 3^3 \\ \Rightarrow \quad 2x &= 3 \\ x &= \frac{3}{2} \\ \therefore \quad \log_9 27 &= \frac{3}{2} \end{aligned}$$

**Class Exercise 2.0**
Evaluate the following Logarithms
(a) $\log_2 4$ (b) $\log_1 01000$ (c) $\log_5 25$
(d) $\log_3 81$ (e) $\log_1 2144$ (f) $\log_2 50.2$
(g) $\log_4 8$ (h) $\log_9 \frac{1}{27}$ (i) $\log_1 000.001$
(j) $\log_{0.2} 25$

**Laws of Logarithms**

1. $\log_a(MN) = \log_a M + \log_a N$
2. $\log_a(M \div N) = \log_a \left(\dfrac{M}{N}\right) = \log_a M - \log_a N$
3. $\log_a M^x = x \log_a M$
4. $\log_a 1 = 0$
5. $\log_a a = 1$

**Example 2.2**
Given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$. Calculate without using table (a) $\log 6$ (b) $\log 9$ (c) $\log 5$
Solution

(a)

$$\begin{aligned} \log 6 = \log(2 \times 3) &= \log 2 + \log 3 \\ &= 0.3010 + 0.4771 \\ &= 0.7781 \end{aligned}$$

(b)

$$\begin{aligned} \log 9 = \log 3^2 &= 2 \log 3 \\ &= 2(0.4771) \\ &= 0.9542 \end{aligned}$$

(c)

$$\begin{aligned} \log 5 = \log \left(\frac{10}{2}\right) &= \log 10 = \log 2 \\ &= 1 - 0.3010 \\ &= 0.6990 \end{aligned}$$

**Class Exercise 2.1**

1. Given that $\log 2 = 0.3010$; $\log 3 = 0.4771$ and $\log 7 = 0.8451$; evaluate the following
   (a) $\log 5$ (b) $\log 8$ (c) $\log 49$ (d) $\log 14$ (e) $\log 35$ (f) $\log 42$
2. Given that $\log 2 = 0.3010$; evaluate $\log 16$ without using table

**Examples 2.3**

1. Simplify $\log 8 + \log 5$
   Solution

$$\begin{aligned} \log 8 + \log 5 &= \log(8) \\ &= \log 40 \end{aligned}$$

2. Simplify $\log 9 \div \log 3$
   Solution

$$\begin{aligned} \log 9 \div \log 3 &= \log \left(\frac{9}{3}\right) \\ &= \log 3 \end{aligned}$$

3. Simplify $\dfrac{2}{3} \log 32$
   Solution

$$\begin{aligned} \frac{2}{3} \log 32 &= \log 32^{\frac{2}{5}} \\ &= \log(2^5)^{\frac{2}{5}} \\ &= \log 2^2 \\ &= \log 4 \end{aligned}$$

**Class Exercise 2.2**

1. Express the following as logarithms of simple numbers
   (a) $\log 3 + \log 4$
   (b) $\log 15 - \log 3$
   (c) $3 \log 5$
   (d) $1 + \log 3$
   (e) $1 - \log 5$
   (f) $\frac{3}{4} \log 16$
2. Simplify the following
   (a) $\log 8 - \log 4$
   (b) $\dfrac{\log 8 - \log 4}{\log 4 - \log 2}$

(c) $\dfrac{\log 4}{\log 2}$

(d) $\dfrac{\log \sqrt{5}}{\log 5}$

(e) $\dfrac{\log 16}{\log 8}$

## 3. SURDS

Numbers such as 5, $2\frac{1}{3}$, 0.37, 0.6, $\sqrt{49}$ can be written as fraction. These kind of numbers are called rational numbers. because we can write them as
$5 = \frac{5}{1}$; $2\frac{1}{3} = \frac{7}{3}$; $0.37 = \frac{37}{100}$; $0.6 = \frac{6}{10}$; $\sqrt{49} = \pm\frac{7}{1}$

They are numbers that cannot be written as exact fraction. An example is the $\pi$ where

$$\pi = 3.141592...$$

These kind of numbers are referred to as Irrational Numbers
Other examples of irrational numbers are $\sqrt{2} = 1.414...$, $\sqrt{3} = 1.7320...$ etc
Irrational numbers in the form

$$a\sqrt{b}$$

where $b$ is not a perfect square is called a Surd.
Examples of surds are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{10}$, $\sqrt{12}$, $\sqrt{13}$ etc

**Laws of Surd**

1. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Note that
$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ ;$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

**Simplification of Surds**
Simplification of surds involves making the number under the square root sign to be as small as possible. This is achieved by expressing the number under the square root sign as a product of two factors, one of which is a perfect square

**Example 3.0**
Simplify (a) $\sqrt{45}$ (b) $3\sqrt{50}$
Solution

(a)

$$\begin{aligned}\sqrt{45} = \sqrt{9 \times 5} &= \sqrt{9} \times \sqrt{5}\\ &= 3 \times \sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

(b)

$$\begin{aligned}3\sqrt{50} = 3 \times \sqrt{25 \times 2} &= 3 \times 5 \times \sqrt{2}\\ &= 15\sqrt{2}\end{aligned}$$

**Class Exercise 3.0**
Simplify the following
(a) $\sqrt{20}$ (b) $\sqrt{32}$ (c) $\sqrt{48}$ (d) $\sqrt{75}$ (e) $\sqrt{72}$ (f) $\sqrt{24}$ (g) $\sqrt{200}$ (h) $\sqrt{150}$ (i) $\sqrt{98}$ (j) $\sqrt{84}$

**Example 3.1**
Express $2\sqrt{6}$ as the square root of a single number
Solution
$2\sqrt{6} = \sqrt{4} \times \sqrt{6} = \sqrt{4 \times 6} = \sqrt{24}$

**Class Exercise 3.1**
Express the following as the square root of a single number
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2}$ (d) $5\sqrt{7}$ (e) $3\sqrt{10}$ (f) $2\sqrt{7}$ (g) $4\sqrt{6}$ (h) $3\sqrt{8}$ (i) $2\sqrt{11}$ (j) $5\sqrt{5}$

**Like Surds**
Two or more surds are said to be like surds if the numbers under the square root sign are the same. Example; $\sqrt{2}$, $2\sqrt{2}$, $\frac{1}{5}\sqrt{2}$ are like surds.

**Addition/Subtraction of Surds**
Two or more surds can be added/subtracted only if they are like surds. Remember, before adding/subtracting two more surds, the individual surds should first be simplified.

**Example 3.2**
Simplify $5\sqrt{2} - 2\sqrt{2}$
Solution
$5\sqrt{2} - 2\sqrt{2} = (5-2)\sqrt{2} = 3\sqrt{2}$
**Example 3.3**
Simplify $\frac{2}{3}\sqrt{5} - 4\sqrt{5}$
Solution

$$\begin{aligned}\frac{2}{3}\sqrt{5} - 4\sqrt{5} &= \left(\frac{2}{3} - 4\right)\sqrt{5}\\ &= \frac{2-12}{3}\sqrt{5}\\ &= -\frac{10}{3}\sqrt{5}\end{aligned}$$

**Example 3.4**
Simplify $\sqrt{12} + \sqrt{3}$
Solution

$$\begin{aligned}\sqrt{12} + \sqrt{3} &= 2\sqrt{3} + \sqrt{3}\\ &= 3\sqrt{3}\end{aligned}$$

**Class Exercise 3.2**
Simplify the following

1. $3\sqrt{2} - \sqrt{18}$
2. $\sqrt{175} - 4\sqrt{7}$
3. $2\sqrt{8} - 3\sqrt{32} + 4\sqrt{50}$
4. $2\sqrt{54} + \sqrt{24} - \sqrt{216}$
5. $3\sqrt{125} - 5\sqrt{20} + 3\sqrt{80}$
6. $\sqrt{60} - \sqrt{375} + \sqrt{135}$
7. $2\sqrt{135} - 2\sqrt{60} + \sqrt{15} - \sqrt{240}$

8. $6+\sqrt{27}+\sqrt{75}$
9. $\sqrt{52}-\sqrt{117}+4\sqrt{13}$
10. $\sqrt{224}-\sqrt{126}-\sqrt{56}$

**Multiplication of Surds**
When two or more surds are multiplied together, they should first be simplified, if possible. Then multiply whole number with whole numbers and surds with surds. Also remember that

$$\sqrt{a}\times\sqrt{a}=a$$

**Example 3.5**
Simplify $\sqrt{27}\times\sqrt{50}$
Solution

$$\begin{aligned}\sqrt{27}\times\sqrt{50}&=3\sqrt{3}\times 5\sqrt{2}\\&=3\times 5\times\sqrt{3}\times\sqrt{2}\\&=15\times\sqrt{3\times 2}=15\sqrt{6}\end{aligned}$$

**Class Exercise 3.3**
Simplify the following

1. $\sqrt{5}\times\sqrt{10}$
2. $\sqrt{8}\times\sqrt{2}$
3. $\sqrt{12}\times\sqrt{3}$
4. $\sqrt{30}\times\sqrt{5}$
5. $\sqrt{32}\times\sqrt{12}$
6. $(\sqrt{3})^5$
7. $(2\sqrt{7})^2$
8. $\sqrt{5}\times\sqrt{24}\times\sqrt{30}$
9. $\sqrt{6}\times\sqrt{8}\times\sqrt{10}\times\sqrt{12}$
10. $(2\sqrt{3})^3$

**Fractional Surds**
These are fraction that contains surd either in the numerator or denominator or both. Examples are $\frac{\sqrt{2}}{3}$, $\frac{5}{\sqrt{7}}$, $\frac{3\sqrt{8}}{2\sqrt{7}}$

**Conjugate of a Surd $a\sqrt{b}$**
The conjugate of a surd $a\sqrt{b}$ is another surd in which their multiplication results in a rational number. Generally the conjugate of $a\sqrt{b}$ is $\sqrt{b}$. For Example the conjugate of $3\sqrt{2}$ is $\sqrt{2}$ because

$$\begin{aligned}3\sqrt{3}\times\sqrt{2}&=3\times\sqrt{2}\times\sqrt{2}\\&=3\times 2\\&=6\end{aligned}$$

**Rationalization of Denominator**
This simply means to convert the denominator of a fractional surd into a rational number. To do this, we multiply the numerator and denominator of the fraction by the conjugate of the denominator.
**Example 3.6**
Rationalize $\frac{6}{\sqrt{3}}$
Solution

$$\begin{aligned}\frac{6}{\sqrt{3}}&=\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\\&=\frac{6\sqrt{3}}{3}\\&=2\sqrt{3}\end{aligned}$$

**Example 3.7**
Rationalize $\frac{7}{\sqrt{18}}$
Solution

$$\begin{aligned}\frac{7}{\sqrt{18}}&=\frac{7}{3\sqrt{2}}\\&=\frac{7}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{7\sqrt{2}}{3(2)}\\&=\frac{7\sqrt{2}}{6}\end{aligned}$$

**Example 3.8**
Rationalize $\frac{\sqrt{5}}{\sqrt{2}}$
Solution

$$\begin{aligned}\frac{\sqrt{5}}{\sqrt{2}}&=\frac{\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{\sqrt{10}}{2}\end{aligned}$$

**Class Exercise 3.4**
Simplify the following by rationalizing the denominators

1. $\frac{2}{\sqrt{2}}$
2. $\frac{6}{\sqrt{2}}$
3. $\frac{4}{\sqrt{8}}$
4. $\frac{15}{\sqrt{3}}$
5. $\frac{2\sqrt{3}}{\sqrt{6}}$
6. $\frac{30}{\sqrt{75}}$
7. $\frac{30}{\sqrt{72}}$
8. $\frac{3\sqrt{2}}{\sqrt{10}}$
9. $\frac{\sqrt{3}\times\sqrt{18}\times\sqrt{39}}{\sqrt{24}\times\sqrt{26}}$

**Binomial Surds**
An expression may contain two terms, which cannot be simplified further. For eg, $6-\sqrt{5}$, $3\sqrt{2}+\sqrt{3}$, $2\sqrt{3}+4$. If one or both of the terms contain a surd, we call this a binomial surd expression.

**Multiplication of Binomial Surds**
When multiplying two binomial surds, use the normal algebraic expansion

$$(a+b)(c+d) = ac+ad+bc+bd$$

**Example 3.9**
Expand and simplify $(3\sqrt{5}+2)(\sqrt{5}+3)$
Solution

$$\begin{aligned}(3\sqrt{5}+2)(\sqrt{5}+3) &= 3(5)+9\sqrt{5}+2\sqrt{5}+6\\ &= 15+11\sqrt{5}+6\\ &= 21+11\sqrt{5}\end{aligned}$$

**Example 3.10**
Expand and simplify $2\sqrt{5}(3\sqrt{5}-2\sqrt{2})$
Solution

$$\begin{aligned}2\sqrt{5}(3\sqrt{5}-2\sqrt{2}) &= 6(5)-4\sqrt{10}\\ &= 30-4\sqrt{10}\end{aligned}$$

**Example 3.11**
Expand and simplify $(2\sqrt{2}+\sqrt{5})^2$
Solution

$$\begin{aligned}(2\sqrt{2}+\sqrt{5})^2 &= (2\sqrt{2}+\sqrt{5})(2\sqrt{2}+\sqrt{5})\\ &= 4(2)+2\sqrt{10}+2\sqrt{10}+5\\ &= 8+4\sqrt{10}+5\\ &= 13+4\sqrt{10}\end{aligned}$$

**Class Exercise 3.5**
Simplify the following

1. $\sqrt{2}(\sqrt{2}+\sqrt{6})$
2. $(\sqrt{5}+\sqrt{15})(2\sqrt{3}-1)$
3. $(\sqrt{6}+2\sqrt{3})^2$
4. $(3\sqrt{2}-\sqrt{5})^2$

**Conjugate of a binomial surd**
The conjugate of a binomial surd $a+\sqrt{d}$ is $a-\sqrt{d}$ and vice versa. **Example 3.12**
Simplify the following by rationalizing the denominators

(a) $\dfrac{2}{3\sqrt{5}+4}$

(b) $\dfrac{6}{2\sqrt{2}-1}$

(c) $\dfrac{2\sqrt{3}+2}{2\sqrt{3}-2}$

Solution

(a)

$$\begin{aligned}\frac{2}{3\sqrt{5}+4} &= \frac{2}{3\sqrt{5}+4}\times\frac{3\sqrt{5}-4}{3\sqrt{5}-4}\\ &= \frac{2(3\sqrt{5}-4)}{(3\sqrt{5}+4)(3\sqrt{5}-4)}\\ &= \frac{6\sqrt{5}-8}{9(5)-12\sqrt{5}+12\sqrt{5}-16}\\ &= \frac{6\sqrt{5}-8}{45-6}\\ &= \frac{6\sqrt{5}-8}{29}\end{aligned}$$

(b)

$$\begin{aligned}\frac{6}{2\sqrt{2}-1} &= \frac{6}{2\sqrt{2}-1}\times\frac{2\sqrt{2}+1}{2\sqrt{2}+1}\\ &= \frac{6(2\sqrt{2}+1)}{(2\sqrt{2}-1)(2\sqrt{2}+1)}\\ &= \frac{12\sqrt{2}+6}{4(2)+2\sqrt{2}-2\sqrt{2}-1}\\ &= \frac{12\sqrt{2}+6}{8-1}\\ &= \frac{12\sqrt{2}+6}{7}\end{aligned}$$

(c)

**Class Exercise 3.6**
Simplify the following by rationalizing the denominator

1. $\dfrac{1}{2-\sqrt{3}}$
2. $\dfrac{4}{3+\sqrt{7}}$
3. $\dfrac{\sqrt{5}}{\sqrt{15}-\sqrt{10}}$
4. $\dfrac{3\sqrt{7}}{5-\sqrt{7}}$
5. $\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
6. $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$
7. $\left(\dfrac{\sqrt{3}\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)^2$